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Development of new mathematical methods and algorithms for verifying the adequacy of mathematical models of objects based on data from a natural experiment to determine the functional stability area

Abstract: The relevance of the development of applied mathematical modelling, which includes numerical methods and software packages in its problem area, its importance for the entire economic activity of the country as a whole, is due to the intensive digitalisation and computerisation of all technological chains of production processes. The integration of production support and various databases, as well as all parts of production and their effective management, require the development of comprehensive research of mathematical methods for modelling production processes. To date, mathematical modelling is applied to calculations of the financial stability point function, which does not fully reflect the variability of the predicted consequences, and consequently, the set of measures to preserve this stability. Due to the complication of production and economic relations, the need for modeling and calculating the area of financial stability, i.e., a set of marginal and non-marginal indicators, under which the economic condition of the enterprise will be considered to be acceptably stable, is actualised. The scientific problem is that mathematical modelling of production and economic processes does not provide for a wide variability (set) of indicators of financial stability as an area, which prevents flexivity in the economic activity of the enterprise. The scientific novelty of the work consists in the development of a method and algorithm for determining the financial stability area of an economic entity. The purpose of the study was to create a mathematical apparatus for calculating the financial stability of an enterprise. In the course of the study, the works of leading scientists and researchers in mathematical modelling and business processing, as well as the works of the authors of the article in this field were used. The authors presented a methodology for the development of quantitative indicators and, based on it, a methodology for mathematical modelling of calculating the financial stability area as a mathematical system that includes all eight main coefficients accepted as parameters of financial stability, and considers the limits that correspond to the economic indicators of the stability of the enterprise.

Keywords: financial stability area, mathematical modelling, business process, algorithmisation, automatisation of modelling calculations, financial stability of the enterprise.

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Разработка новых математических методов и алгоритмов проверки адекватности математических моделей объектов на основе данных натурального эксперимента по определению области функциональной стабильности

Аннотация: Актуальность развития прикладного математического моделирования, включающая в свою проблемную область численные методы и комплексы программ, его значение для всей хозяйственной деятельности страны в целом, обусловлена интенсивной цифровизацией и компьютеризацией всех технологических цепочек производственных процессов. Интеграция производственного обеспечения и различных баз данных, а также всех звеньев производства и их эффективного управления, требуют развития комплексных исследований математических методов моделирования производственных процессов. На сегодняшний день математическое моделирование применяется к расчётам функции точки финансовой стабильности, что не отражает в полной мере вариативность прогнозируемых последствий, а следовательно, и комплекса мероприятий по сохранению данной стабильности. В связи с усложнением производственных и экономических отношений актуализируется необходимость моделирования и расчёта области финансовой стабильности, т.е., совокупности предельных и непредельных показателей, при которых экономическое состояние предприятие будет считаться допустимо стабильным. Научной проблемой является то, что математическое моделирование производственных и экономических процессов не предусматривает широкую вариативность (множество) показателей финансовой стабильности как области, что препятствует флексивности в экономической деятельности предприятия. Научная новизна работы заключается в разработке метода и алгоритма определения области финансовой стабильности экономического субъекта. Целью исследования было создание математического аппарата вычисления области финансовой стабильности предприятия. В ходе работы были использованы труды ведущих учёных и исследователей в области математического моделирования и бизнес-процессинга, а также работы авторов статьи в данной области. Авторы представили методологию разработки количественных показателей и на базе неё методологию математического моделирования расчёта области финансовой стабильности (устойчивости) как математической системы, которая включает в себя все восемь основных коэффициентов, принятых как параметры финансовой стабильности, и учитывает пределы, которые соответствуют экономическим показателям стабильности предприятия.

Ключевые слова: область финансовой стабильности, математическое моделирование, бизнес-процесс, алгоритмизация, автоматизация расчётов моделирования, финансовая стабильность предприятия.

Introduction

The relevance of the development of applied mathematical modelling, which includes numerical methods and software packages in its problem area, its importance for the entire economic activity of the country as a whole, is due to the intensive digitalisation and computerisation of all technological chains of production processes. The integration of production support and various databases, as well as all parts of production and their effective management, require the development of comprehensive research of mathematical methods for modelling production processes.

Microeconomic processes, as well as business processes, are a basic component of the life cycle of any enterprise. To date, mathematical modelling is applied to calculations of the financial stability point function, which does not fully reflect the variability of the predicted consequences, and consequently, the set of measures to preserve this stability. Due to the complication of production and economic relations, the need for modeling and calculating the area of financial stability is actualized, i.e., a set of marginal and non-marginal indicators under which the economic condition of the enterprise will be considered to be permissible stable.

The scientific problem is that mathematical modelling of production and economic processes does not provide for a wide variability (set) of indicators of financial stability as an area, which prevents flexivity in the economic activity of the enterprise.

The object of the study was mathematical modelling of sets of production and economic indicators.

The subject of the study was methods and algorithms of mathematical modelling of production and economic indicators.

The purpose of the study was to create a mathematical apparatus for calculating the financial stability of an enterprise.

To achieve the purpose, it is necessary to solve the following study tasks:

- develop a model of algorithmisation and automation of calculations modelling the financial stability of the enterprise;
- design a methodology for the development of quantitative indicators;
- develop a methodology for mathematical modelling of calculating the financial stability area (FSA).

General scientific logical, historical, statistical, comparative methods, mathematical modelling, data analysis, and generalisation were used to achieve the purpose and solve problems in the course of the study.

The study's scientific novelty is the development of a method and algorithm for determining the financial stability area of an economic entity.

The study's theoretical significance is the development of a method for defining a new concept in the economic analysis of an enterprise – the field of financial stability.

The study's practical significance is the development of a new mathematical method and algorithm for verifying the adequacy of mathematical models of objects based on natural experiment data.

In the course of the work, the works of leading scientists and researchers in the field of mathematical modelling and business processing, e.g., L. Zadeh (*1965*), G.C. Chow (*1997*), C.W. Churchman (*1963*), D.N. Gujarati (*1992*; *1995*), M. Harzallah (*2007*), I., Hofacker and R. Vetschera (*2001*), K.-Y. Jeong (*2008*), M. Koubarakis (*2002*), K.I. Kurpayanidi (*2019*), S.G. Powell, M. Schwaninger and C. Trimble (*2001*), A.E. Teshabaev (*2018*), K. Vergidis and A. Tiwari (*2008*), L. Whitman and B. Huff (*1997*), as well as the works of the authors of the article in this field were used (*Buychik, 2021a*; *Buychik, 2021b*; *Komissarov, 2021a*; *Komissarov, 2021b*).

Development of models for algorithmisation and automatisation of calculations in financial stability of the enterprise

The first part of the article describes the development of models for algorithmisation and automatisation of calculations for modelling the financial stability of an enterprise and algorithms for their verification in EPC and IDEF0 notations, as well as using the ABC methodology (*Buychik, 2021a*).

Modelling in the EPC notation (event chain of processes) characterises the process of automating calculations as a set of sequential measures for the production of the final product – determining the indicators of the financial stability of the enterprise (*Figure 1*). Based on the model executed in EPC notation and tested on the experimental site of the enterprise, the microlevel processes of each stage of the EPC model were modeled in IDEF notation (*Business Process Model and Notation*) to create an algorithm of actions and minimise the risks of producing erroneous intermediate indicators, which are later used to obtain the area of financial stability (*Figure 2*).

To develop models of algorithmisation and automatisation of calculations for modeling the financial stability area, it is necessary to create an algorithm for providing initial (resource) data from the company's balance sheet.

1. The financial autonomy coefficient shows the share of equity and the reserve value of assets and is calculated by dividing equity by total assets. Consequently, the constituent components of equity in the company's balance sheet are:

- 1310 "Authorised Capital (share capital, authorised capital, contributions of associates)";
- 1320 "Own Shares Repurchased from Shareholders";
- 1340 "Revaluation of Non-Current Assets";
- 1350 "Additional Capital (without revaluation)";
- 1360 "Reserve Capital";
- 1370 "Retained Earnings (uncovered loss)".

Using IDEF notation, equity can be represented in the form of a diagram (*Figure 3*). Total assets are recorded on the left side of the balance sheet – its asset. The total amount of non-current assets is indicated in line 1100, current assets – in line 1200. Their amount on the balance reflects line 1600. Therefore, using IDEF notation, aggregate assets can also be represented in the form of a diagram (*Figure 4*). Thus, the algorithmisation and automatisation model of calculating the coefficient of financial autonomy can be represented by a consolidated algebraic scheme (*Figure 5*).

2. The coefficient of own and borrowed funds represents the share of borrowed funds in total sources of financing. The algorithm for calculating equity is shown in part 1. In the company's balance sheet, line 1410 "Credits, loans (long-term liabilities)" and line 1510 of the same name "Credits, loans (short-term liabilities)" are provided for reflecting borrowed funds. Therefore, using IDEF notation, borrowed funds can be represented in the form of a diagram (*Figure 6*). Thus, the algorithmisation and automatisation model of calculating the coefficient of own and borrowed funds can be represented by a consolidated algebraic scheme (*Figure 7*).

3. The coefficient of availability of own working capital provides an assessment of the availability of the company's own funds for financial support of current activities. In mathematical terms, the calculation involves dividing the difference between equity and non-current assets by working capital. The algorithm for calculating equity is shown in part 1. The indicator of non-current assets in the company's balance sheet is reflected in nine lines. Therefore, using IDEF notation, non-current assets can be represented schematically (*Figure 8*). Current assets include six lines of the balance sheet. Therefore, using IDEF notation, current assets can also be represented schematically (*Figure 9*). Thus, the algorithmisation and automatisation model of calculating the coefficient of provision with own working capital can be represented by a consolidated algebraic scheme (*Figure 10*).

4. The financial stability coefficient provides a generalised or generalised analysis of the main sources of financing of the company's assets and is calculated by dividing the amount of own and long-term borrowed funds by the currency of the organisation's operations. The indicator of own funds (assets) is presented in part 1. The indicator of long-term borrowed funds is determined by line 1410. The indicator of the currency balance is determined by lines 1600 "Currency of the Balance of Assets" and 1700 "Currency of the Liabilities Balance". Therefore, using IDEF notation, the currency balance can be represented schematically (*Figure 11*). Thus, the algorithmisation and automatisation model of calculating the financial stability coefficient can be represented by a consolidated algebraic scheme (*Figure 12*).

5. The maneuverability coefficient of equity represents the level of total liquidity of the financial assets of the enterprise and represents the private difference of equity and non-current assets for the same equity. Thus, the algorithmisation and automatisation model of calculating the maneuverability coefficient of equity can be represented by a consolidated algebraic scheme (*Figure 13*).

6. The coefficient of the degree of solvency of a legal entity reflects the coefficient of the ability of the enterprise to pay its current obligations. The coefficient and is calculated by dividing the amount of current liabilities by the average monthly revenue. Thus, the algorithmisation and automatisation model for calculating the solvency coefficient can be represented by an algebraic scheme (*Figure 14*).

7. The short-term debt ratio shows the share of short-term sources of borrowed funds that generate risks to the financial stability of a legal entity. The coefficient is a quotient of short-term

and total borrowed funds. Therefore, using IDEF notation, the short-term debt coefficient can be represented by an algebraic scheme (*Figure 15*).

8. The current liquidity coefficient shows the ability of an enterprise to direct current assets to repay its own short-term liabilities. The coefficient is the ratio of current assets and borrowed funds. Thus, the algorithmisation and automatisation model of the calculation of the current liquidity coefficient can be represented by an algebraic scheme (*Figure 16*).

Thus, the first part of the article presents the results of modeling the algorithmisation of all eight coefficients for calculating the financial stability of the enterprise. In the generalised version, the general algorithm can be represented by a business process diagram (*Figure 17*).

Methodology for the development of quantitative indicators

The second part of the article provides a methodology for the development of quantitative indicators that will be used to model the financial stability area.

The ABC (triplicity of indicators) methodology is used in modern financial management to determine extreme and median indicators, which are later used in production or strategic planning, as well as calculations in complex mathematical models of decision-making and obtaining results under uncertainty.

From the economic side of modeling innovative production and economic projects, which include the calculation of the financial stability area, the most important condition is the presentation of a model that can be determined during the relevant analysis. The analysis of the financial stability of the enterprise was performed using the calculation of the coefficients of eight multicomponent key indicators:

- the coefficient of autonomy (K_A),
- the coefficient of own and borrowed funds (K_{OBF}),
- the coefficient of the enterprise's own working capital (K_{EOWC}),
- the coefficient of financial stability (K_{FS}),
- the coefficient of equity's maneuverability (K_{EM}),
- the coefficient of solvency (K_S),
- the coefficient of short-term debt (K_{STD}),
- the coefficient of current liquidity (K_{CL}).

Consequently, the methodology for the development of quantitative indicators took into account the above indicators:

$$N = \sum K \times n_a, \qquad (1)$$

there:

N is sum of indicators,

$\sum K = \{K_A, K_{OBF}, K_{EOWC}, K_{FS}, K_{EM}, K_S. K_{STD}, K_{CL}\}$

n_a is number of alternatives.

Since eight indicators are used in the calculation, the following formula is used:

$$N = 8 \times n_a. \qquad (2)$$

In the course of modelling, the analysis of the inclusion of calculations of a set of key indicators for determining the financial stability of an enterprise in a set of criteria for choosing an alternative was performed. The criterion of indifference was excluded from the criteria, since the study was conducted considering the definition of a set of indicators of financial stability, in which an alternative with the maximum average result is calculated, which by definition is included in financial stability.

The remaining four criteria were used to construct the calculation of the set of alternatives:

$$M = 4N, \qquad (3)$$

there:

M is number of alternatives,

N is the sum of the indicators of each alternative.

The alternative assumes the presence of at least two options; therefore, the following formula is applicable in an expanded form:

$$M = 4 \times 8 \times n_a = 32 \times \sum_{i=2}^{n} n_a. \qquad (4)$$

Thus, if one scenario (a set of indicators) obtained under the conditions of modelling one situation, i.e., one set of parameters, is included in the calculations, at least 64 solutions are presented as a set of points of financial stability within the relevant area.

When considering the variation of the indicators of the share of equity and the reserve of the value of assets that issue as a private coefficient of financial autonomy, at least six options are generated, which, in turn, determine the appropriate number of alternatives. As part of the decision-making process to determine the point of financial stability, four criteria are applied for the financial stability area under conditions of uncertainty, which increases the number of alternatives:

$$M = 4 \times 8 \times n_a \times 6 = 192 \times \sum_{i=2}^{n} n_a. \qquad (5)$$

As a result of the use of a variety of options for only one of the financial stability coefficients, when it is limited exclusively to tenths, a variation that is a multiple of 192 when considering each subsequent alternative, arises.

When obtaining such a large set of indicators, it is proposed to introduce the triplicity principle of final indicators (ABC methodology) into the algorithm at each stage of calculations, i.e., the output of *maximax*, *minimin*, and *median* (*mid*) indicators. Thus, each of the eight coefficients in the final form is represented as three indicators, forming a more specific and optimal set of 24 final indicators of financial stability, forming areas of financial stability:

$$M = 8 \times n_a \times 3 = 24n_a. \qquad (6)$$

In the course of a natural experiment at the enterprise of the Viaduct LLC, specialising in the production of crackers, biscuits, and other breadcrumbs, the production of flour confectionery, cakes, pastries, pies, and biscuits intended for long-term storage, the minimum, median and optimal coefficients were used for calculations (*Table 1*).

To describe the development of quantitative indicators, it is necessary to present all the coefficients of financial stability of an enterprise in the form of mathematical expressions (*Table 2*).

Thus, the calculation of the total coefficients in the mathematical model can be represented as follows:

$$\sum k_{min} = k_{afs}^{min} + k_{rfs}^{min} + k_{pfs}^{min} + k_{jfs}^{min} + k_{mfs}^{min} + k_{sfs}^{min} + k_{dfs}^{min} + k_{lfs}^{min}, \qquad (7)$$

$$\sum k_{mid} = k_{afs}^{mid} + k_{rfs}^{mid} + k_{pfs}^{mid} + k_{jfs}^{mid} + k_{mfs}^{mid} + k_{sfs}^{mid} + k_{dfs}^{mid} + k_{lfs}^{mid}, \qquad (8)$$

$$\sum k_{max} = k_{afs}^{max} + k_{rfs}^{max} + k_{pfs}^{max} + k_{jfs}^{max} + k_{mfs}^{max} + k_{sfs}^{max} + k_{dfs}^{max} + k_{lfs}^{max}. \quad (9)$$

Based on the total indicator of the minimum coefficients, the average indicator will be calculated as follows:

$$M_{min} = \frac{\sum k_{min}}{8}. \qquad (10)$$

Therefore, according to the economic indicators of financial stability, the average minimum indicator will be as follows:

$$M_{min} = \frac{\sum k_{min}}{8} = \frac{0.5 + 0.5 + 0.2 + 0.8 + 0.3 + 0.5 + 0 + 1}{8} = 0.475.$$

Based on the total indicator of the median coefficients, the average indicator will be calculated as follows:

$$M_{mid} = \frac{\sum k_{mid}}{n}. \qquad (11)$$

Therefore, according to the economic indicators of financial stability, the average median indicator will be as follows:

$$M_{mid} = \frac{\sum k_{mid}}{n} = \frac{0.6 + 0.6 + 0.3 + 0.9 + 0.45 + 0.6 + 0.1 + 1}{8} = 0.569.$$

Based on the total indicator of the maximum coefficients, the average indicator will be calculated as follows:

$$M_{max} = \frac{\sum k_{max}}{n}. \qquad (12)$$

Therefore, according to the economic indicators of financial stability, the average maximum indicator will be as follows:

$$M_{max} = \frac{\sum k_{max}}{n} = \frac{0.7 + 0.7 + 0.4 + 1 + 0.6 + 0.7 + 0.2 + 1}{8} = 0.663.$$

Accordingly, the financial stability area of the Viaduct LLC enterprise is within the range of indicators from 0.475 to 0.663, which will contain 24 indicators.

To calculate the limits of the conditions of the financial stability area, the sums of the extreme corresponding coefficients of the indicators and the remaining two averaged indicators are used.

To calculate the lowest extreme indicator of the financial stability area, it is used the formula:

$$M_{minimin} = \frac{k_{min}^{min} + M_{mid} + M_{max}}{3}. \qquad (13)$$

To calculate the extreme highest indicator of the financial stability area, it is used the formula:

$$M_{maximax} = \frac{k_{max}^{max} + M_{min} + M_{mid}}{3}. \qquad (14)$$

Consequently, further calculations of the extreme conditions of the area, based on the minimum coefficient indicators in each group of indicators, were made:

$$M_{minimin} = \frac{k_{min}^{min} + M_{mid} + M_{max}}{n} = \frac{0 + 0.569 + 0.663}{3} = 0.411,$$

$$M_{maximax} = \frac{k_{max}^{max} + M_{min} + M_{mid}}{n} = \frac{1 + 0.475 + 0.569}{3} = 0.681.$$

Consequently, the Viaduct LLC's financial stability area will be a set of 192 indicators that fall within the limits of indicators from 0.411 to 0.681.

These indicators and limits of the set are fully confirmed by the accounting financial stability of Viaduct LLC for the fiscal year 2021.

Thus, the second part of the article provides a methodology for the development of quantitative indicators that will be used to model the financial stability area. In the course of modelling, the analysis of including calculations of a set of key indicators for determining the financial stability of an enterprise in a set of criteria for choosing an alternative was performed. The criterion of indifference was excluded from the criteria, since the study was conducted considering the definition of a set of indicators of financial stability, in which an alternative with the maximum average result, which by definition is included in the financial stability area, is calculated. When obtaining a large set of indicators, it is proposed to introduce the principle of triplicity of final indicators into the algorithm at each stage of calculations. Thus, in the final form, each of the eight coefficients is represented as three indicators, forming a more specific and optimal set of 24 final indicators of financial stability, forming the financial stability areas. The approbation of this methodology on the materials of the economic indicators of Viaduct LLC for the 2021 fiscal year confirmed its effectiveness and compliance with the financial analysis of the enterprise according to the accounting documentation.

Methodology of mathematical modelling of calculating the financial stability area

The third part of the article presents the methodology of mathematical modeling of calculating the financial stability area as a mathematical system.

When mathematically modelling a mathematical system for calculating the financial stability area, it is necessary to consider the areas of optimal variation in the indicators of each initially separately presented component, presented as a normative value, therefore, is the mathematical limits of the indicators of each component.

For the mathematical representation of economic indicators, the ratio of which the economic coefficients of financial stability are calculated, it is necessary to determine their mathematical designations (*Table 3*).

Based on this, the authors present the coefficients of the main components of the system and the formulas for their calculation with mathematical limits.

1. The autonomy coefficient shows the share of equity and the reserve value of assets and is calculated by dividing equity by total assets:

$$K_A = \frac{EQ}{TA}, \qquad (15)$$

there:

K_A is the coefficient of autonomy,

EQ is the indicator of equity,

TA is the indicator of total assets.

Therefore, the calculation of the coefficient can be represented mathematically:

$$k_{afs} = \frac{i_{eq}}{i_{ta}}, \qquad (16)$$

there:

k_{afs} is the coefficient of autonomy,

i_{eq} is the indicator of equity,

i_{ta} is the indicator of total assets.

Based on the regulatory limits, this formula can be represented as $0.5 \leq K_A \leq 0.7$, or $0.5 \leq k_{afs} \leq 0.7$, i.e., $k_{afs}(0.5, 0.7)$.

2. The coefficient of own and borrowed funds represents the share of borrowed funds in total sources of financing:

$$K_{OBF} = \frac{BF}{FS}, \qquad (17)$$

there:

K_{OBF} is the coefficient of own and borrowed funds,

BF is the indicator of borrowed funds,

FS is the indicator of the source of funding.

Therefore, the calculation of the coefficient can be represented mathematically:

$$k_{rfs} = \frac{i_{bf}}{i_{fs}}, \qquad (18)$$

there:

k_{rfs} is the coefficient of own and borrowed funds,

i_{bf} is the indicator of borrowed funds,

i_{fs} is the indicator of the source of funding.

Based on the regulatory limits, this formula can be represented as $0.5 \leq K_{OBF} \leq 0.7$, or $0.5 \leq k_{rfs} \leq 0.7$, i.e., $k_{rfs}(0.5, 0.7)$.

3. The coefficient of the enterprise's own working capital provides an assessment of the availability of the company's own funds for financial support of current activities. In mathematical terms, the calculation involves dividing the difference between equity and non-current assets by working capital:

$$K_{EOWC} = \frac{EQ - nCA}{CA}, \qquad (19)$$

there:

K_{EOWC} is the coefficient of the enterprise's own working capital,

EQ is the indicator of equity,

nCA is the indicator of non-current assets,

CA is the indicator of current assets.

Therefore, the calculation of the coefficient can be represented mathematically:

$$k_{pfs} = \frac{i_{ec} - i_{nca}}{i_{ca}}, \qquad (20)$$

there:

k_{pfs} is the coefficient of the enterprise's own working capital,

i_{ec} is the indicator of equity,

i_{nca} is the indicator of non-current assets,

i_{ca} is the indicator of current assets.

Based on the regulatory limits, this formula can be represented as $0.2 \leq K_{EOWC} \leq 0.4$, or $0.2 \leq k_{pfs} \leq 0.4$, i.e., $k_{pfs}(0.2, 0.4)$.

4. The coefficient of financial stability provides a generalised analysis of the main sources of financing of the company's assets. It is calculated by dividing the amount of own and long-term borrowed funds by the currency of the enterprise's operations:

$$K_{FS} = \frac{FS + LBF}{CB}, \qquad (21)$$

there:

K_{FS} is the coefficient of financial stability,

FS is indicator of the source of funding,

LBF is indicator of long-term borrowed funds,

CB is indicator of currency balance.

Therefore, the calculation of the coefficient can be represented mathematically:

$$k_{jfs} = \frac{i_{fs} + i_{lbf}}{i_{cb}}, \qquad (22)$$

there:

k_{jfs} is the coefficient of financial stability,

i_{fs} is the indicator of the source of funding,

i_{lbf} is the indicator of long-term borrowed funds,

i_{cb} is the indicator of currency balance.

Based on the regulatory limits, this formula can be represented as $0.8 \leq K_{FS} \leq 1$, or $0.8 \leq k_{jfs} \leq 1$, i.e., $k_{jfs}(0.8, 1)$.

5. The coefficient of equity's maneuverability represents the level of total liquidity of the financial assets of the enterprise and represents the private difference of equity and non-current assets on the same equity:

$$K_{EM} = \frac{EQ - nCA}{EQ}, \qquad (23)$$

there:

K_{EM} is the coefficient of equity's maneuverability,

EQ is the indicator of equity,

nCA is the indicator of non-current assets.

Therefore, the calculation of the coefficient can be represented mathematically:

$$k_{mfs} = \frac{i_{eq} - i_{nca}}{i_{eq}}, \qquad (24)$$

there:

k_{mfs} is the coefficient of equity's maneuverability,

i_{eq} is the indicator of equity,

i_{nca} is the indicator of non-current assets.

Based on the regulatory limits, this formula can be represented as $0.3 \leq K_{EM} \leq 0.6$, or $0.3 \leq k_{mfs} \leq 0.6$, i.e., $k_{mfs}(0.3, 0.6)$.

6. The degree of solvency of a legal entity reflects the coefficient of the enterprise's ability to pay its current obligations. The coefficient and is calculated by dividing the amount of current liabilities by the average monthly revenue:

$$K_S = \frac{CLA}{AMR}, \qquad (25)$$

there:

K_S is the coefficient of solvency,
CLA is the indicator of the amount of current liabilities,
AMR is the indicator of average monthly revenue.

Therefore, the calculation of the coefficient can be represented mathematically:

$$k_{sfs} = \frac{i_{cla}}{i_{amr}}, \qquad (26)$$

there:
k_{sfs} is the coefficient of solvency,
i_{cla} is the indicator of the amount of current liabilities,
i_{amr} is the indicator of average monthly revenue.

Based on the regulatory limits, this formula can be represented as $0.5 \leq K_S \leq 0.7$, or $0.5 \leq k_{sfs} \leq 0.7$, i.e., $k_{sfs}(0.5, 0.7)$.

7. The coefficient of short-term debt shows the share of short-term sources of borrowed funds that generate risks to the financial stability of a legal entity. The coefficient is a quotient of short-term and total borrowed funds:

$$K_{STD} = \frac{CLA}{TBR}, \qquad (27)$$

there:
K_{STD} is the coefficient of short-term debt,
CLA is the indicator of the amount of current liabilities,
TBR is the indicator of total borrowed funds.

Therefore, the calculation of the coefficient can be represented mathematically:

$$k_{dfs} = \frac{i_{cla}}{i_{tbr}}, \qquad (28)$$

there:
k_{dfs} is the coefficient of short-term debt,
i_{cla} is the indicator of the amount of current liabilities,
i_{tbr} is the indicator of total borrowed funds.

Based on the regulatory limits, this formula can be represented as $0 \leq K_{STD} \leq 0.2$, or $0 \leq k_{dfs} \leq 0.2$, i.e., $k_{dfs}(0, 0.2)$.

8. The coefficient of current liquidity shows the ability of an enterprise to direct current assets to repay its own short-term liabilities. The coefficient is the ratio of current assets and borrowed funds:

$$K_{CL} = \frac{CA}{CLA}, \qquad (29)$$

there:
K_{CL} is the coefficient of current liquidity,
CA is the indicator of current assets,
CLA is the indicator of the amount of current liabilities.

Therefore, the calculation of the coefficient can be represented mathematically:

$$k_{lfs} = \frac{i_{ca}}{i_{cla}}, \qquad (30)$$

there:
k_{lfs} is the coefficient of current liquidity,

i_{ca} is the indicator of current assets,
i_{cla} is the indicator of the amount of current liabilities.

Based on the regulatory limits, this formula can be represented as $K_{CL} = 1$, or $k_{lfs} = 1$.

Thus, according to the financial limits of the coefficients, the mathematical limits of the sets of indicators are also established (*Table 4*).

Since all eight coefficients of a single process are used in the mathematical modelling of this system, their interpretation can be presented in the form of indicators of related k_{fs} coefficients (*Table 2*).

Several patterns are observed in the comparative analysis of the limits.

1. Similarity of coefficient indicators:
$k_{afs} \sim k_{rfs} \sim k_{sfs}$, or $k_{afs}(0.5, 0.7)$, $k_{rfs}(0.5, 0.7)$ and $k_{sfs}(0.5, 0.7)$.

Therefore, the sum of these coefficients in the model can be represented by the arithmetic mean of the set:

$$k_{fs}^{1} = \frac{k_{afs} + k_{rfs} + k_{sfs}}{3}. \qquad (31)$$

It follows from the calculations that

$$k_{fs}^{1} \in (0.5, 0.7).$$

2. The reversibility of the coefficients is $k_{jfs} + k_{dfs} = 1$, because $k_{jfs}(0.8, 1)$ and $k_{dfs}(0, 0.2)$ despite the fact that these coefficients have oppositely directed vectors of values at the enterprise.

Therefore, the sum of these coefficients in the model can be represented by a separate coefficient:

$$k'_{fs} = k_{jfs} + k_{dfs} = 1. \qquad (32)$$

$$k'_{fs} = 1.$$

3. The similarity of the indicators of the initial and synthesised coefficients: $k'_{fs} \sim k_{lfs}$ because $k_{fs}^{2} = 1$ and $k_{lfs} = 1$.

Therefore, the sum of these coefficients in the model can be represented by an arithmetic mean equal to 1:

$$k_{fs}^{2} = \frac{k'_{fs}}{k_{lfs}} = 1. \quad (33)$$

$$k_{fs}^{2} = 1.$$

In expanded form, this formula can be represented as:

$$k_{fs}^{2} = \frac{k_{jfs} + k_{dfs}}{k_{lfs}} = 1. \qquad (34)$$

$$k_{fs}^{2} = 1.$$

4. Compositeness of coefficient indicators: $k_{pfs}(0.2, 0.4)$ and $k_{mfs}(0.3, 0.6)$.

Therefore, the sum of the numerical indicators of these coefficients in the model will represent clearly defined limits from 0.5 to 1:

$$k_{fs}^{3} = k_{pfs} + k_{mfs} = (0.5, 1). \qquad (35)$$

$$k_{fs}^{3} \in (0.5, 1).$$

Thus, to calculate the financial stability area as a system of indicators, the mathematical model will be presented as follows:

$$k_{fs} = k_{fs}^1 + k_{fs}^2 + k_{fs}^3 = \frac{k_{afs} + k_{rfs} + k_{sfs}}{3} + \frac{k_{jfs} + k_{dfs}}{k_{lfs}} + k_{sfs} + k_{mfs}, \qquad (36)$$

i.e., $k_{fs} \in (2, 2.7)$.

When rounding k_{fs} to tenths, a set of indicators of the financial stability area equal to eight is determined. The whole set will determine the options for the stability of the system.

To verify k_{fs} in the system of the financial stability area, another mathematical interpretation is applicable.

Since $k_{fs}^3 \geq k_{fs}^1$ not less than 2, but not more than 4, then:

$$2 \leq \frac{k_{fs}^3}{k_{fs}^1} \leq 4. \qquad (37)$$

Therefore, $k_{fs}^{3/1}(2, 4)$.

At the same time, the mathematical model of the financial stability area can be represented in the following expression:

$$k_{fs} = \frac{k_{fs}^{3/1}}{k_{fs}^2} = \frac{k_{fs}^3}{k_{fs}^1 * k_{fs}^2}, \qquad (38)$$

i.e., $k_{fs}(2, 4)$.

When rounding k_{fs} to tenths, a set of indicators of the financial stability area equal to 11 is determined. The whole set will determine the options for the stability of the system.

Thus, the full mathematical model of the system for calculating the financial stability area can be represented as follows:

$$\begin{cases} k_{fs} = k_{fs}^1 + k_{fs}^2 + k_{fs}^3 \\ 2 \leq \dfrac{k_{fs}^3}{k_{fs}^1} \leq 4 \end{cases}. \qquad (39)$$

Based on the Viaduct LLC enterprise, a natural experiment was performed on the calculations of the financial stability area. The indicators of the enterprise's financial stability coefficients for the 4th quarter of 2021 were applied (*Table 5*).

1. The calculation of similar indicators was made:

$$k_{fsq1}^1 = \frac{0.59 + 0.62 + 0.55}{3} = 0.59 \in (0.5, 0.7).$$

$$k_{fsq2}^1 = \frac{0.51 + 0.66 + 0.58}{3} = 0.58 \in (0.5, 0.7).$$

$$k_{fsq3}^1 = \frac{0.61 + 0.69 + 0.63}{3} = 0.64 \in (0.5, 0.7).$$

$$k_{fsq4}^1 = \frac{0.55 + 0.58 + 0.60}{3} = 0.58 \in (0.5, 0.7).$$

2. The calculation and verification of the reversibility of the coefficients was performed:

$k'_{fsq1} = 0.80 + 0.20 = 1.$

$k'_{fsq2} = 0.91 + 0.09 = 1.$

$k'_{fsq3} = 0.92 + 0.08 = 1.$

$k'_{fsq4} = 0.86 + 0.14 = 1.$

All quarterly reversible coefficients corresponded to 1.

3. The similarity of the indicators of the initial and synthesised coefficients of reversible quarterly indicators and the current liquidity coefficient was confirmed:

$k_{fs^{q1}}^{2} = \frac{1}{1} = 1.$

$k_{fs^{q2}}^{2} = \frac{1}{1.01} = 1.$

$k_{fs^{q3}}^{2} = \frac{1}{1.01} = 1.$

$k_{fs^{q4}}^{2} = \frac{1}{0.99} = 1.$

4. The compositeness of the coefficients k_{pfs} and k_{mfs} is calculated:

$k_{fs^{q1}}^{3} = 0.21 + 0.35 = 0.56 \in (0.5, 1),$

$k_{fs^{q2}}^{3} = 0.28 + 0.37 = 0.65 \in (0.5, 1),$

$k_{fs^{q3}}^{3} = 0.33 + 0.39 = 0.72 \in (0.5, 1),$

$k_{fs^{q4}}^{3} = 0.31 + 0.36 = 0.67 \in (0.5, 1).$

By the next stage of the algorithm for calculating the financial stability area as a system of the indicators of mathematical model, calculations were performed:

$k_{fs^{q1}} = k_{fs^{q1}}^{1} + k_{fs^{q1}}^{2} + k_{fs^{q1}}^{3} = 0.59 + 1 + 0.56 = 2.15 \in (2, 2.7),$

$k_{fs^{q2}} = k_{fs^{q2}}^{1} + k_{fs^{q2}}^{2} + k_{fs^{q2}}^{3} = 0.58 + 1 + 0.65 = 2.23 \in (2, 2.7),$

$k_{fs^{q3}} = k_{fs^{q3}}^{1} + k_{fs^{q3}}^{2} + k_{fs^{q3}}^{3} = 0.64 + 1 + 0.72 = 2.36 \in (2, 2.7),$

$k_{fs^{q4}} = k_{fs^{q4}}^{1} + k_{fs^{q4}}^{2} + k_{fs^{q4}}^{3} = 0.58 + 1 + 0.67 = 2.25 \in (2, 2.7).$

To verify quarterly k_{fs} in the financial stability area system, a mathematical interpretation was applied:

$$2 \le \frac{k_{fs^q}^{3}}{k_{fs^q}^{1}} \le 4.$$

1st quarter – 2.15 : 0.59 = 3.644,
2nd quarter – 2.23 : 0.58 = 3.845,
3rd quarter – 2.36 : 0.64 = 3.688,
4th quarter – 2.25 : 0.58 = 3.880.

Consequently, the quarterly parameters of the financial stability coefficients fully comply with the conditions of the model of the second variant:

$$\begin{cases} k_{fs} = k_{fs}^{1} + k_{fs}^{2} + k_{fs}^{3} \\ 2 \le \frac{k_{fs}^{3}}{k_{fs}^{1}} \le 4 \end{cases}.$$

Since the results of the calculations of mathematical modelling of the financial stability area fully correspond to the quarterly accounting statements on the economic stability of the Viaduct LLC, this model can be considered practically applicable and relevant for determining the financial stability area.

Thus, the developed methodology of mathematical modelling of calculating the financial stability area as a mathematical system, it includes all eight main coefficients accepted as parameters of financial stability, and includes the limits that correspond to the economic indicators of the stability of the enterprise. On the basis of the Viaduct LLC enterprise, a natural experiment was performed on the calculations of the financial stability area. The indicators of the enterprise's financial stability coefficients for the 4th quarter of 2021 were applied. The simulation results showed the limits of the complex parameters of financial stability corresponding to the desired parameters of "financial stability". In full, the mathematical model of the system for calculating the financial stability area can be presented as follows:

$$\begin{cases} k_{fs} = k_{fs}^1 + k_{fs}^2 + k_{fs}^3 \\ 2 \le \frac{k_{fs}^3}{k_{fs}^1} \le 4 \end{cases}.$$

Discussion

In the course of the study, modelling and algorithmisation of business processes was implemented to calculate the financial stability area for a small enterprise. In this regard, it is necessary to develop this mathematical approach to modelling financial stability for large enterprises and holdings to digitalise this model to the global level of application.

Conclusion

Thus, within the framework of the study, the analysis of the development of numerical methods and algorithms of the processes of functioning of the enterprise was performed.

In the first part, the authors describe the development of algorithmisation models and automatisation of calculations for modelling the enterprise's financial stability and algorithms for their verification in EPC and IDEF0 notations, as well as using the ABC methodology.

In the second part, the authors provide a methodology for the development of quantitative indicators that will be used to model the financial stability area. In the course of modelling, the analysis to include calculations of a set of key indicators for determining the enterprise's financial stability in a set of criteria for choosing an alternative was performed. The criterion of indifference was excluded from the criteria, because the study was conducted including the definition of a set of indicators of financial stability, in which an alternative with the maximum average result included in financial stability by definition is calculated. When obtaining a large set of indicators, it is proposed to introduce the triplicity principle of final indicators into the algorithm at each stage of calculations. Thus, each of the eight coefficients in the final form is represented as three indicators, forming a more specific and optimal set of 24 final indicators of financial stability, forming financial stability areas. The approbation of this methodology on the materials of the economic indicators of Viaduct LLC for the 2021 financial year confirmed its effectiveness and compliance with the financial analysis of the enterprise according to the accounting documentation.

In the third part, the authors present the developed methodology of mathematical modelling of calculating the financial stability area as a mathematical system that includes all eight main coefficients accepted as parameters of financial stability, and includes the limits that correspond to the economic indicators of the stability of the enterprise. Based on the Viaduct LLC, a natural experiment was performed on the calculations of the financial stability area. The indicators of the enterprise's financial stability coefficients for the 4th quarter of 2021 were applied. The simulation results showed the limits of the complex parameters of financial stability corresponding to the desired parameters of "financial stability". Variants of the mathematical model for calculating the financial stability area have been successfully tested on the basis of the Viaduct LLC.

* * *

References:

Business Process Model and Notation (BPMN). Version 2.0. https://www.omg.org/spec/BPMN/2.0/PDF

Buychik, A. (2021a). Basic principles application of the ABC methodology in human resource management at the enterprise. *Actual Issues of Management Development. European Scientific e-Journal*, *12*(6), 24-32.

Buychik, A. (2021b). Updating the parameters of the development of effective economic thought to motivate society to finance innovative activities. *Economy at the Crossroads of Time. European Scientific e-Journal*, *10*(4), 7-16.

Chow, G. C. (1997). *Dynamic economics: Optimization by the Lagrange method.* New York: Oxford University Press.

Churchman, C. W. (1963). *Thinking for Decisions: Deductive Quantitative Methods*. Chicago, Illinois: Science Research Associates.

Gujarati, D. N. (1992). *Essentials of econometrics*. New York: McGraw-Hill.

Gujarati, D. N. (1995). *Basic Econometrics*. New York: McGraw-Hill.

Harzallah, M. (2007). Incorporating IDEF3 into the Unified Enterprise Modelling Language. *Proceedings of the 2007 Eleventh International IEEE EDOC Conference Workshop*, 133-140.

Hofacker, I., & Vetschera, R. (2001). Algorithmical approaches to business process design. *Computer & Operations Research*, *28*, 1253-1275.

Jeong, K.-Y. (2008). Integration of queuing network and IDEF3 for business process analysis. *Business Process Management Journal*, *14*(4), 471-482.

Komissarov, P. V. (2021a). Comprehensive assessment of the base of mathematical modelling of production business processes. *Actual Issues of Management Development. European Scientific e-Journal*, *14*(8), 7-23.

Komissarov, P. V. (2021b). Determination of the centric rate of the economic stability domain for manufacturing enterprises. *Economy at the Crossroads of Time. European Scientific e-Journal*, *10*(4), 27-36.

Koubarakis, M. (2002). A formal framework for business process modelling and design. *Information Systems*, *27*, 299-319.

Kurpayanidi, K. I. (2019). Theoretical basis of management of innovative activity of industrial corporation. *ISJ Theoretical & Applied Science*, *69*(1), 7-14. (In Russian)

Powell, S. G., Schwaninger, M., & Trimble, C. (2001). Measurement and control of business processes. *System Dynamics Review*, *17*(1), 63-91.

Teshabaev, A. E. (2018). The methodological approaches to management improving for modern companies. *Scientific Technical Journal*, 22, 108-115. (In Russian)

Vergidis, K., & Tiwari, A. (2008). Business process analysis and optimization: beyond reengineering. *IEEE Transactions on Systems, Man, Cybernatics. Part C: Application and Reviews*, 1-14.

Whitman, L., & Huff, B. (1997). Structured models and dynamic systems analysis: The integration of the IDEF0/IDEF3 modeling methods and discrete event simulation. *Proceedings of Simulation Conference*, 518-524.

Zadeh, L. (1965). Fuzzy Sets. *Information & Control*, *8*, 338-353.

Appendix

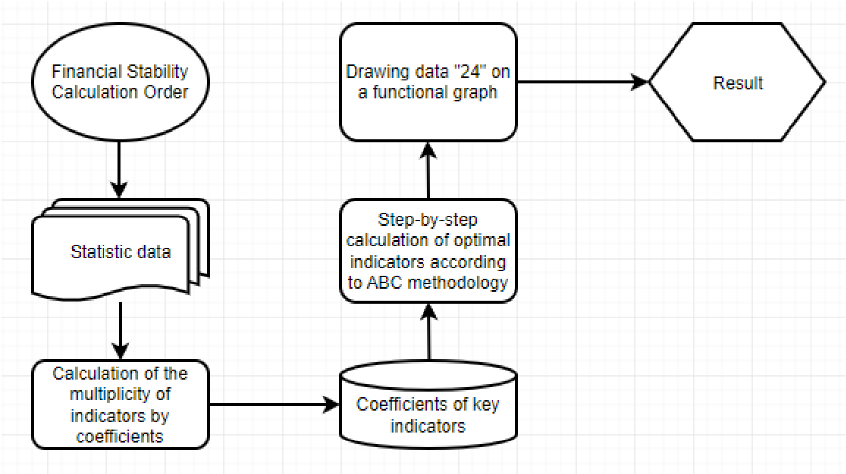

Figure 1. Example of creating a business process algorithm for generating the financial stability area of an enterprise in EPC notation

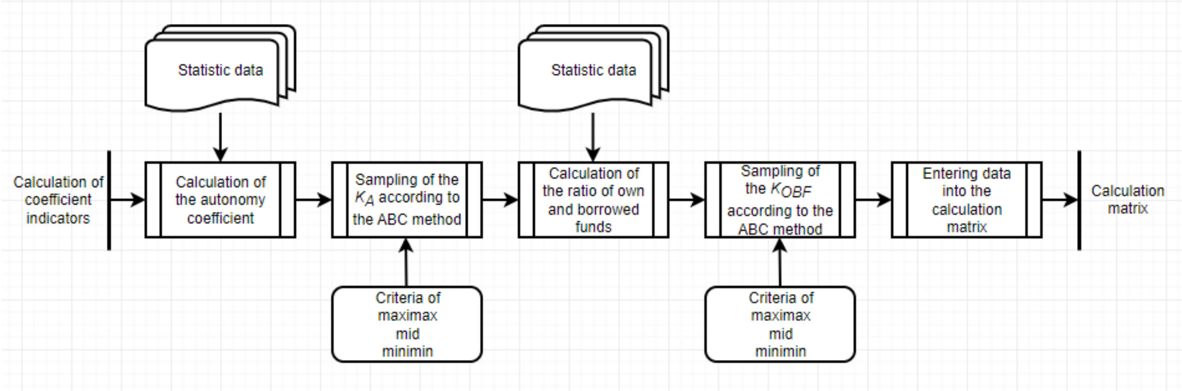

Figure 2. An example of creating a business process of algorithm for generating a calculated matrix of data coefficients of enterprise's key indicators in IDEF notation

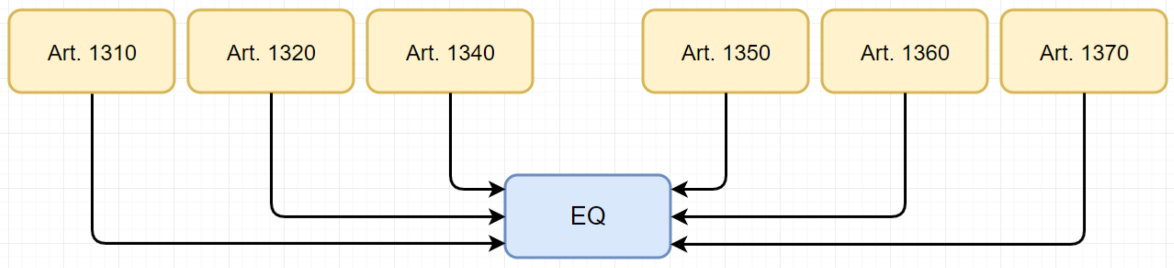

Figure 3. Algorithm for calculating equity

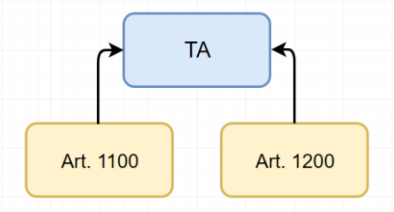

Figure 4. Algorithm for calculating total assets

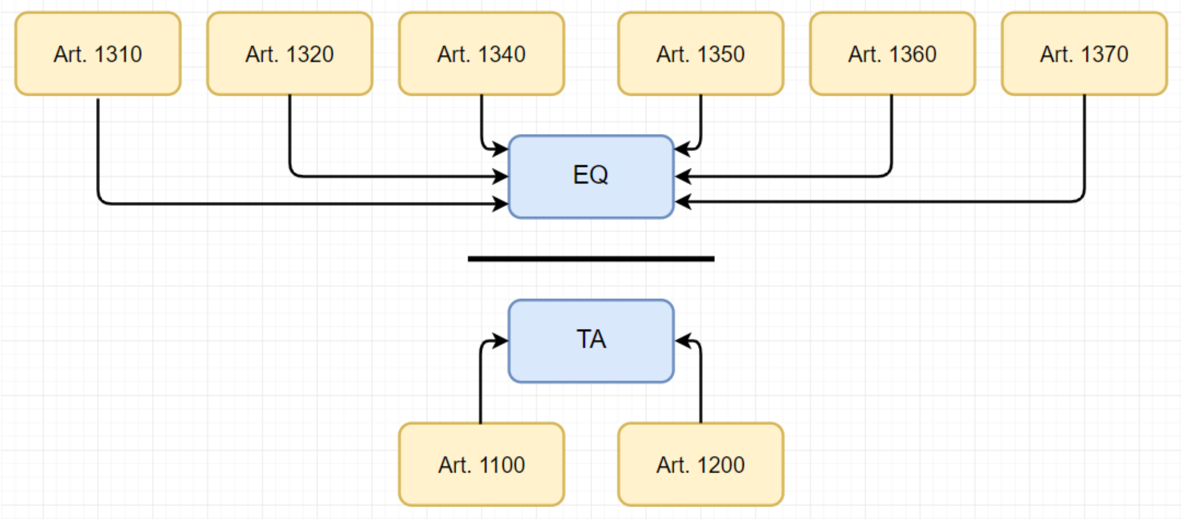

Figure 5. Algorithmisation and automatisation model for calculating the financial autonomy coefficient

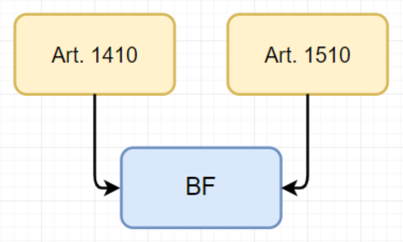

Figure 6. Algorithm for calculating borrowed funds

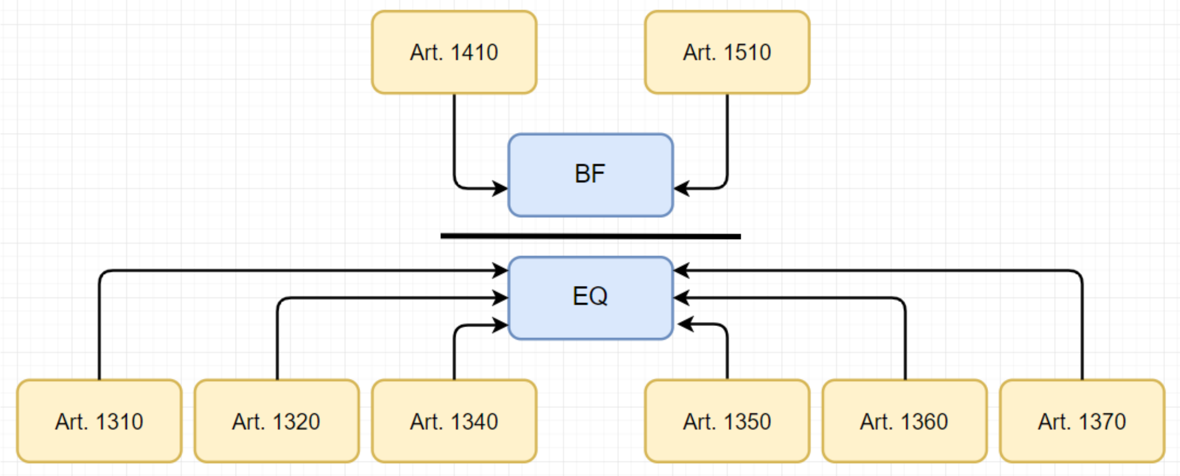

Figure 7. Algorithmisation and automatisation model for calculating the coefficient of own and borrowed funds

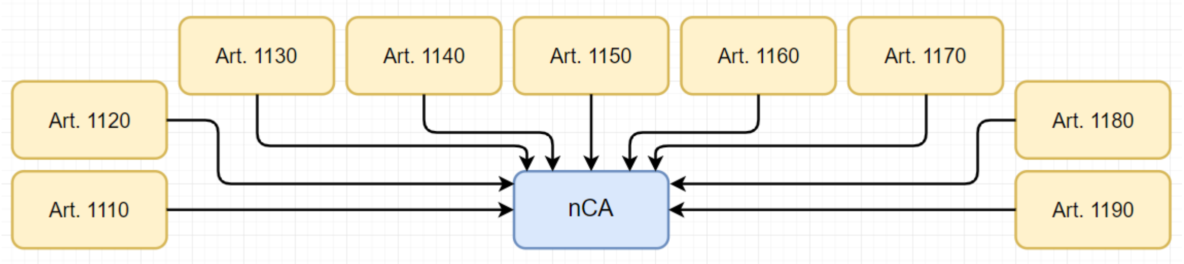

Figure 8. Algorithm for calculating non-current assets

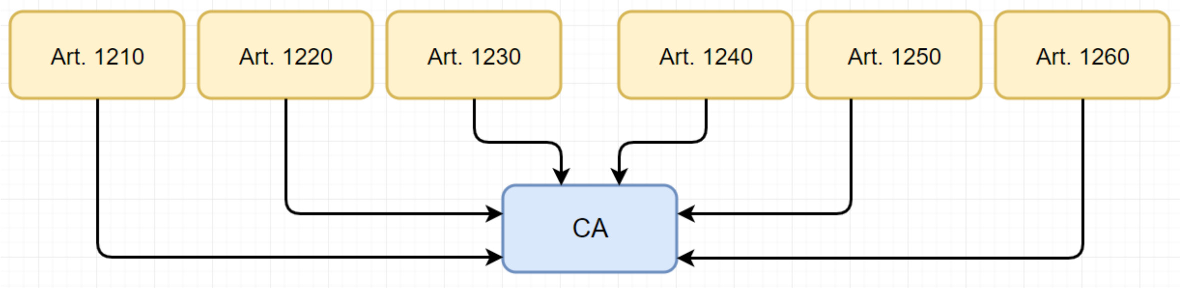

Figure 9. Algorithm for calculating current assets

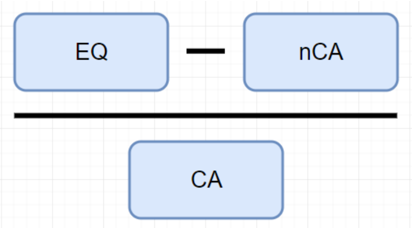

Figure 10. Algorithmisation and automation model for calculating the coefficient of provision with own working capital

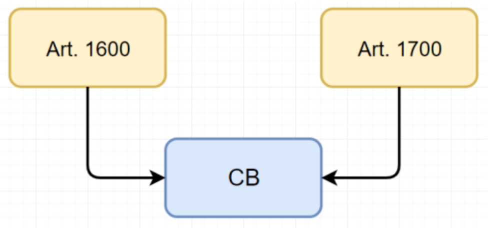

Figure 11. Algorithm for calculating the currency balance

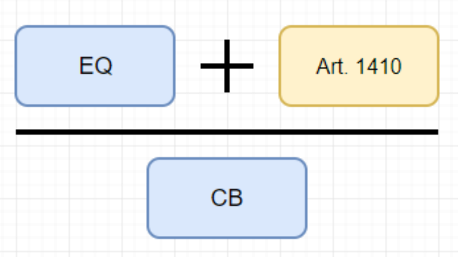

Figure 12. Algorithmisation and automatisation model for calculating the financial stability coefficient

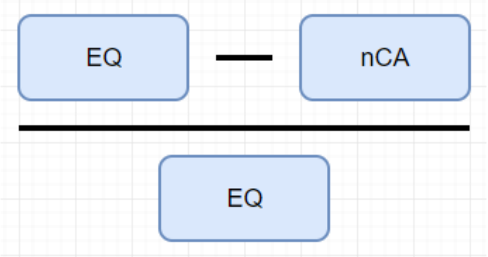

Figure 13. Algorithmisation and automation model for calculating the coefficient of equity maneuverability

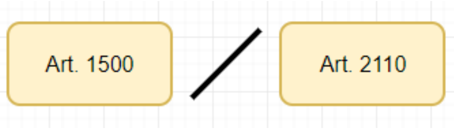

Figure 14. Algorithmisation and automation model for calculating the coefficient of solvency

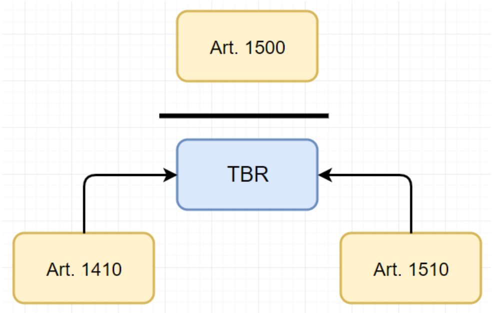

Figure 15. Algorithmisation and automation model for calculating the coefficient of short-term debt

Figure 16. Algorithmisation and automatisation model of calculation of the coefficient of current liquidity

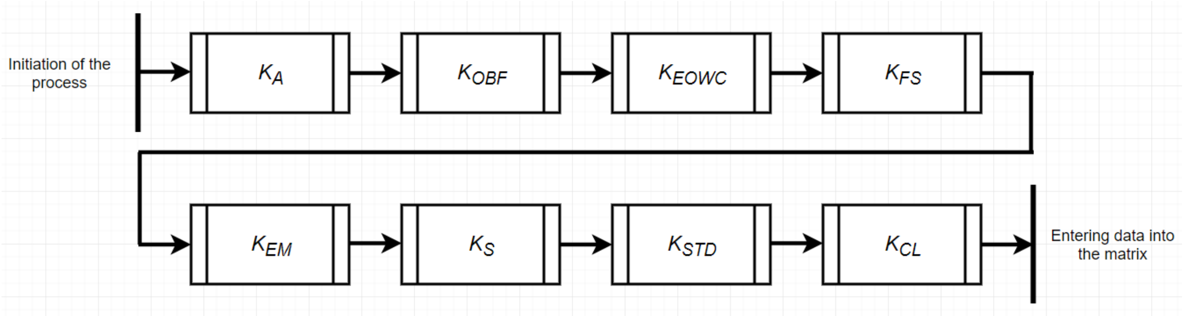

Figure 17. Business process diagram of the algorithmisation model for all eight financial stability calculation coefficients

Table 1. Indicators of economic coefficients of the Viaduct LLC enterprise by the end of 2021

Coefficient		Mini	Mid	Maxi
The coefficient of autonomy	K_A	0.5	0.6	0.7
The coefficient of own and borrowed funds	K_{OBF}	0.5	0.6	0.7
The coefficient of the enterprise's own working capital	K_{EOWC}	0.2	0.3	0.4
The coefficient of financial stability	K_{FS}	0.8	0.9	1
The coefficient of equity's maneuverability	K_{EM}	0.3	0.45	0.6
The coefficient of solvency	K_S	0.5	0.6	0.7
The coefficient of short-term debt	K_{STD}	0	0.1	0.2
The coefficient of current liquidity	K_{CL}	1	1	1

Table 2. Definition of mathematical notation for the financial stability area system (FSA)

Coefficient	Economic notation	Mathematical notation
The coefficient of autonomy	K_A	k_{afs}
The coefficient of own and borrowed funds	K_{OBF}	k_{rfs}
The coefficient of the enterprise's own working capital	K_{EOWC}	k_{pfs}
The coefficient of financial stability	K_{FS}	k_{jfs}
The coefficient of equity's maneuverability	K_{EM}	k_{mfs}
The coefficient of solvency	K_S	k_{sfs}
The coefficient of short-term debt	K_{STD}	k_{dfs}
The coefficient of current liquidity	K_{CL}	k_{lfs}

Table 3. Definition of mathematical designations of economic indicators for calculating financial stability coefficients

Indicator	Economic notation	Mathematical notation
Indicator of equity	EQ	i_{eq}
Indicator of total assets	TA	i_{ta}
Indicator of borrowed funds	BF	i_{bf}
Indicator of the source of funding	FS	i_{fs}
Indicator of non-current assets	nCA	i_{nca}
Indicator of current assets	CA	i_{ca}
Indicator of long-term borrowed funds	LBF	i_{lbf}
Indicator of currency balance	CB	i_{cb}
Indicator of the amount of current liabilities	CLA	i_{cla}
Indicator of average monthly revenue	AMR	i_{amr}
Indicator of total borrowed funds	TBR	i_{tbr}

Table 4. Limits of efficiency indicators of coefficients

Coefficient	Economic notation	Mathematical notation	Mini	Maxi	Limits
The coefficient of autonomy	K_A	k_{afs}	0.5	0.7	(0.5, 0.7)
The coefficient of own and borrowed funds	K_{OBF}	k_{rfs}	0.5	0.7	(0.5, 0.7)
The coefficient of the enterprise's own working capital	K_{EOWC}	k_{pfs}	0.2	0.4	(0.2, 0.4)
The coefficient of financial stability	K_{FS}	k_{jfs}	0.8	1	(0.8, 1)
The coefficient of equity's maneuverability	K_{EM}	k_{mfs}	0.3	0.6	(0.3, 0.6)
The coefficient of solvency	K_S	k_{sfs}	0.5	0.7	(0.5, 0.7)
The coefficient of short-term debt	K_{STD}	k_{dfs}	0	0.2	(0, 0.2)
The coefficient of current liquidity	K_{CL}	k_{lfs}	1	1	(1)

Table 5. Quarterly indicators of financial stability coefficients of Viaduct LLC for the fiscal year 2021

Economic notation	Mathematical notation	Quarter 1	Quarter 2	Quarter 3	Quarter 4
K_{A}	k_{afs}	0.59	0.51	0.61	0.55
K_{OBF}	k_{rfs}	0.62	0.66	0.69	0.58
K_{EOWC}	k_{pfs}	0.21	0.28	0.33	0.31
K_{FS}	k_{jfs}	0.80	0.91	0.92	0.86
K_{EM}	k_{mfs}	0.35	0.37	0.39	0.36
K_{S}	k_{sfs}	0.55	0.58	0.63	0.60
K_{STD}	k_{dfs}	0.20	0.09	0.08	0.14
K_{CL}	k_{lfs}	1	1.01	1.01	0.99